COURSE: SPECIAL TOPICS IN MATHEMATICS: LINEAR ALGEBRA AND CONVEX OPTIMIZATION
COURSE DELIVERY METHOD: CLASSROOM INSTRUCTION Face-To-Face, THEORETICAL-PRACTICAL
CLASSROOM WORKLOAD: 3 HOURS / WEEK
PREREQUISITES: NONE
CREDITS: 3
INSTRUCTOR: WILBER ACUÑA-BRAVO

SUMMARY OF THE COURSE

This course reviews the basics of practical Linear Algebra and introduces the student to the topic of optimization models and convex optimization. The first part, Linear Algebra, covers the general notions of this area required as a basis for the second part; this is, from basic operations up to solving linear equations, least-squares methods, among others. The second part of the course is devoted to the introduction of convex optimization, providing the student with the mathematical machinery required for solving everyday engineering problems, such as machine learning, statistics, decision-making and control systems.

GENERAL OBJECTIVE

To provide an introduction to the Convex Optimization modelling as well as the linear algebra theory behind the optimization and modelling.

SPECIFIC OBJECTIVES

- To present the basic theory of Linear Algebra required for understanding, modelling and solving most of the problems that arise in engineering
- to give students the training for identify and solve the problems by using the linear algebra machinery
- to present the basic theory of Convex Optimization required for understanding, modelling and solving most of the problems that arise in engineering
- to give students the tools to recognize convex optimization problems that arise in applications, the understanding of how such problems are solved, and some experience in solving them to be further applied in their research.

METHODOLOGY

A mix of lectures, tutorials and individual assignments should be used in the course. Lectures will be given by the teacher where he will introduce the students to the theoretical basis of the linear algebra and convex optimization. Illustrative examples will be presented in order to conceptualize the main results around the subject.

Lectures will be held in Spanish language, but all the material and resources will be written in English.

COURSE CONTENTS:

1. INTRODUCTION
  1.1. Motivating examples
  1.2. Optimization problems
  1.3. Important classes of optimization problems
  1.4. History
2. VECTORS AND FUNCTIONS
  2.1. Vector basics
  2.2. Norms and inner products
  2.3. Projections onto subspaces
  2.4. Functions
3. MATRICES
  3.1. Matrix basics
  3.2. Matrices as linear maps
  3.3. Determinants, eigenvalues, and eigenvectors
  3.4. Matrices with special structure and properties
  3.5. Matrix factorizations
  3.6. Matrix norms
  3.7. Matrix functions
4. SYMMETRIC MATRICES
  4.1. Basics
  4.2. The spectral theorem
  4.3. Spectral decomposition and optimization
  4.4. Positive semidefinite matrices
5. SINGULAR VALUE DECOMPOSITION
  5.1. Singular value decomposition
  5.2. Matrix properties via SVD
  5.3. SVD and optimization
6. LINEAR EQUATIONS AND LEAST SQUARES
  6.1. Motivation and examples
  6.2. The set of solutions of linear equations
  6.3. Least-squares and minimum-norm solutions
  6.4. Solving systems of linear equations and LS problems
  6.5. Sensitivity of solutions
  6.6. Direct and inverse mapping of a unit ball
  6.7. Variants of the least-squares problem
7. CONVEXITY
  7.1. Convex sets

7.2. Convex functions
7.3. Convex problems
7.4. Optimality conditions

8. LINEAR, QUADRATIC, AND GEOMETRIC MODELS
8.1. Unconstrained minimization of quadratic functions
8.2. Geometry of linear and convex quadratic inequalities
8.3. Linear programs
8.4. Quadratic programs
8.5. Modeling with LP and QP
8.6. LS-related quadratic programs
8.7. Geometric programs

9. SECOND-ORDER CONE AND ROBUST MODELS
9.1. Second-order cone programs
9.2. SOCP-representable problems and examples
9.3. Robust optimization models

10. SEMIDEFINITE MODELS
10.1. From linear to conic models
10.2. Linear matrix inequalities
10.3. Semidefinite programs
10.4. Examples of SDP

ACADEMIC ACTIVITIES AND WORKLOAD

CLASSROOM INSTRUCTION WORKLOAD [HOURS]		OFF-CLASSROOM WORKLOAD [HOURS]		TOTAL WORKLOAD [HOURS]	CREDITS
THEORETICAL	PRACTICE	THEORETICAL	PRACTICE		
48	0	96	0	144	3

EVALUACIÓN Y PORCENTAJES

NUMBER	%	COMPONENTS
1	50	Weekly homework assignments (100%)
2	25	First part exam (50%)
3	25	Final exam (50%)

HARDWARE & SOFTWARE RESOURCES

Software for simulation Matlab

TEXTBOOKS & MATERIALS

The class is based on a book by Giuseppe Calafiore and Laurent El Ghaoui.

- **Giuseppe Calafiore and Laurent El Ghaoui. *Optimization Models*. Cambridge University Press. 2014. ISBN 978-1-107-05087-7**

The following books are useful as additional reference texts:

- Stephen Boyd and Lieven Vandenberghe. *Introduction to Applied Linear Algebra – Vectors, Matrices, and Least Squares*. Cambridge University Press, 2018. ISBN: 978-1-31651896-0

- Stephen Boyd and Lieven Vandenberghe. *Convex Optimization*. Cambridge University Press, 2004. ISBN: 978-0-52183378-3