INTERACTIONS BETWEEN THE THEORY OF REPRESENTATION OF ALGEBRAS, NUMBER THEORY AND COMBINATORICS

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ALTENCOA8-2018
Popayán, Colombia
23 al 27 de julio de 2018

Resumen. In the last few years many mathematicians have pay much attention to some results in the theory of representation of algebras which have a strong relationship with number theory and combinatorics. For instance, R. Schiffler, C.M. Ringel et al have investigated the role of continued fractions, perfect matchings, Catalan numbers and Fibonacci numbers as invariants of some algebras of finite, tame and wild representation type.

In this talk, we present some of these results giving a relationship of some of them with the Vietá's formula ($\frac{\sqrt{2}}{2}\frac{\sqrt{2+\sqrt{2}}}{2}\frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2}\cdots=\frac{2}{\pi}$) and describe how a nice sequence of integer matrix introduced by A.G. Zavadskij to classify indecomposable representations of a generalization of the Kronecker problem (the semilinear Kronecker problem) can be used to solve linear systems of differential equations.

Introducción

The interest for the role of the number theory and combinatorics in the theory of representation of algebras has growth in recent years, perhaps such activity has been encouraged by Ringel et al who coined the term *categorification of an integer sequence* as the process for which numbers in the sequence can be seen as invariants of objects in a category. For example, Ringel and Fahr used the Auslander-Reiten quiver of the 3-kronecker quiver to obtain partition formulas for Fibonacci numbers. Soon afterwards, Ringel proposed to create a OEDF (On-Line Encyclopedia of Dynkin Functions) as the famous OEIS in such a way that it can be possible to encode the different real or integer sequences arising from the Dynkin diagrams. In this setting Catalan numbers are a fundamental tool, for instance, Catalan numbers give the number of antichains in mod Λ_n where Λ_n denotes a Dynkin diagram of type $\mathbb{A}_n$ linearly oriented.

Many integer sequences arise from modules of hereditary artin algebras and the factorization of its elements is a very important problem regarding its categorification. On the other hand, tame algebras as the Kronecker algebra has been a source of many interesting problems in different areas of the Mathematics for example

Key words and phrases. Algebra representation, Categorification, Continued fraction, Cluster algebra, Dynkin algebra, Integer partition, Snake graph, Differential equation.

Aplicaciones.

indecomposable Kronecker modules give solutions to linear systems of differential equations.

We also recall that at the earliest of the first decade of this century Fomin and Zelevinsky introduced cluster algebras since then new research have been arose establishing categorifications of integer sequences. Catalan numbers are a good example of this situation bearing in mind that they give the number of cluster variables of a cluster algebra of Dynkin type $\mathbb{A}$. Recently, R. Schiffler et al, used perfect matchings of some snake graphs to obtain a formula for cluster variables, such results allowed them to establish properties of continued fractions and some results regarding sums of two squares and Markov triples. Markov triples are related with clusters variables of the cluster algebra of the one puncture torus.

In this talk, we will describe these results and the way of how it is possible to use the Gutman energy of Dynkin diagrams to classify algebras, obtain the Vietá's formula, solve system of linear differential equations of order two and give formulas for partitions of integer numbers. [4, 5, 6, 9, 8, 10].

1. Interactions Between The Theory of Representation of Algebras, Number Theory and Combinatorics

The theory of representation of algebras is a great source for applications of many results of number theory and combinatorics, for this reason Ringel proposes to categorify integer and real sequences by interpreting its elements as invariants of objects in a category. For instance, it is easy to see that there is a bijection between Catalan numbers and cluster variables of a cluster algebra of type $\mathbb{A}$.

Ringel's proposal encouraged many mathematicians to work in that direction, on one hand by categorifying non-crossing partitions via cluster algebras or by using the Auslander-Reiten quiver of some path algebras. We recall that A.M. Cañadas et al have used tiled orders and Kronecker modules to categorify sequences A052558 and A016269 this sequence in particular counts the number of two-point antichain in the powerset of an n-element set and it is possible to prove that it has no prime numbers [2, 3, 1]. On the other hand, we recall that a triple of integer numbers (a, b, c) is said to be a Markov triple if $a^2 + b^2 + c^2 = 3abc$, numbers a, b and c are called Markov numbers regarding these numbers in 1913 Frobenius conjectured that *the largest number in a Markov triple determines the other two*, Propp proved that Markov numbers is the number of perfect matchings of the Markov snake graphs, and Schiffler proved that every Markov number is a sum of two squares or the numerator of an even palindromic continued fraction. In this talk, we will describe these results and some calculus arising from solutions of the semilinear Kronecker problem [7, 8, 10].

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